

## Math 307A&B Summer 2004

### REVIEW PROBLEMS ON ALGEBRA, TRIGONOMETRY, AND CALCULUS

These problems cover some things you must know in order not to fail Math 307. You will be tested on this material on Friday June 25.

#### Algebra.

- (1) Write down all values of  $r$  which are solutions of the quadratic equation  $mr^2 + k = 0$  where  $m$  and  $k$  are both positive real numbers.
- (2) Write down all values of  $r$  which are solutions to the quadratic equation  $mr^2 + cr + k = 0$ , where  $m, c$  and  $k$  are real numbers and  $m \neq 0$ .
- (3) Simplify the expression  $\frac{\frac{1}{AB}}{\frac{1}{A} + \frac{1}{B}}$ , where  $A$  and  $B$  are non-zero real numbers.
- (4) Simplify the expression  $\frac{x^{1/3}x^{-1/5}}{x^{-a}}$ , where  $x$  is a positive real number and  $a$  is any real number.
- (5) Solve the following equation for  $K$  in terms of the other quantities.

$$\frac{\frac{1}{K} + a}{\frac{1}{K} + b} = e^{-rt}$$

where  $a$  and  $b$  are constants. Simplify as much as possible.

- (6) Solve the following expression for  $y$ :

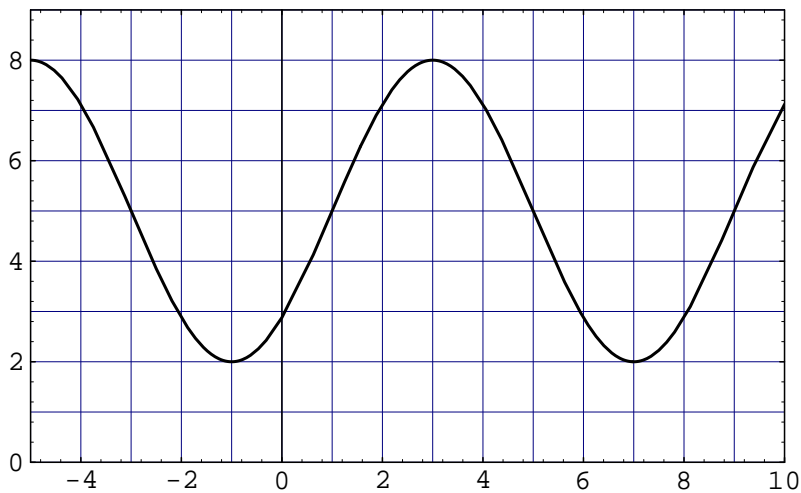
$$\ln(x) - \ln(y) = -2 \ln(y) + ax + b$$

where  $x, a$  and  $b$  are positive real numbers. Simplify as much as possible.

- (7) Express some basic properties of the natural logarithm and its inverse, the exponential function, by completing the following:
  - (a)  $\ln xy = ?$
  - (b)  $e^x e^y = ?$
  - (c)  $e^{\ln x} = ?$
  - (d)  $\ln e^x = ?$
  - (e)  $\ln 1 = ?$
- (8) Find counterexamples to each of the following “identities”; that is, find specific numbers  $x = a, y = b$  such that equality **FAILS** for  $a$  and  $b$ . (For instance, to show that  $(x + y)^2 \neq x^2 + y^2$ , it would suffice to take  $x = 1, y = 1$ , since  $(1 + 1)^2 = 4 \neq 2 = 1^2 + 1^2$ .)
  - (a)  $\ln(x + y) = \ln x + \ln y$  ( $x > 0, y > 0$ ).
  - (b)  $\frac{1}{x} + y = \frac{1}{x} + \frac{1}{y}$  ( $x, y \neq 0$ ).
  - (c)  $\sqrt{x + y} = \sqrt{x} + \sqrt{y}$  ( $x \geq 0, y \geq 0$ ).
  - (d)  $e^{x+y} = e^x + e^y$ .

**Trigonometry.**

- (1) Use the formula for the cosine of the sum of two angles to express  $f(t) = \sqrt{3}\sin(7t) - \cos(7t)$  in the form  $f(t) = a\cos(bt + c)$ , where  $a$ ,  $b$ , and  $c$  are real numbers.
- (2) The graph below is the graph of a function of the form  $y = A\cos(\omega t - \phi) + b$ . Find the specific values of  $A$ ,  $\omega$ ,  $\phi$  and  $b$ .



- (3) Sketch the curves in the plane given by each of the following two parametric equations:

(a) 
$$\begin{cases} x = 4\cos(3t) \\ y = 3\sin(3t) \end{cases}, 0 \leq t \leq \pi/2;$$

(b) 
$$\begin{cases} x = e^{-0.5t}\cos(2\pi t + \pi) \\ y = e^{-0.5t}\sin(2\pi t + \pi) \end{cases}, 0 \leq t \leq 4.$$

**Calculus.** Evaluate each of the following integrals.

You'll need to understand the chain rule for differentiation and a couple of basic methods of integration: substitution (including trigonometric substitutions), integration by parts, and integration of rational functions with quadratic denominator. Remember the necessity of adding a constant of integration for indefinite integrals.

(1) Evaluate each of the following:

$$(a) \int \sin(2\pi t) / \sqrt{2 + \cos(2\pi t)} dt$$

$$(b) \int_0^1 x^2 dx / (4 - 4x + x^2)$$

$$(c) \int_0^1 x^4 dx / (4 + x^2)$$

$$(d) \int x \sin(ax + b) dx$$

$$(e) \int \text{Arctan } x dx$$

$$(f) \int \frac{dx}{\sqrt{x^2 - r^2}}$$

$$(g) \int_0^\infty \frac{(x + 2)dx}{\sqrt{x^2 + 4x + 5}}$$

$$(h) \int \frac{x - a}{x - b} dx$$

$$(i) \int_{-1/2}^{1/2} x e^{2(x+1)} dx$$

$$(j) \int e^{3t} \sin 3t dt$$