CSEP 561 – Capacity

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Topic

- How much digital information can we send over an analog channel?
  - This is the channel capacity

- Two fundamental limits:
  - Nyquist rate (ignores noise)
  - Shannon capacity
Fundamental communication limits

• Given that we can use media to deliver a signal with some strength (S) over some bandwidth (B), how much information can we send?

• Fundamental limits (Nyquist/Shannon) tell us the theoretical maximums that may be achieved.

• Practical schemes for modulating the signal, e.g., QAM, must fall within these limits.
Nyquist Rate (~1924)

- For a noiseless channel with bandwidth B
- Frequencies attenuation limits the rate of symbol transitions

\[ R = 2B \text{ symbols/sec}, \text{ e.g., } 3\text{KHz} \rightarrow 6\text{Ksym/sec} \]

- With V discrete signal levels per symbol:
  \[ R = 2B \cdot \log V \text{ bits/sec}, \text{ e.g., } 3\text{KHz, 4 levels} \rightarrow 12 \text{ Kbps} \]
Taking Noise into Account

- Noise limits how many signal levels we can safely distinguish between
  - $S =$ signal amp., $N =$ noise amp.
  - Really, noise is random described by a mean and variance
  - AWGN = Additive White Gaussian Noise

- The number of bits per symbol depends on the number of distinguishable signal levels
  - E.g, 4 levels implies 2 bits / symbol
Shannon capacity (1948)

\[
\text{Capacity} = \text{Bandwidth} \times \log_2 \left( 1 + \frac{\text{Signal}}{\text{Noise}} \right)
\]


EE: Bandwidth = width of frequency band, measured in Hz
CS: Bandwidth = information carrying capacity, measured in bits/sec
Signal-to-Noise ratio (SNR)

- Signal-to-noise ratio = (wanted) signal / (unwanted)noise
  - SNR = S/N, so substituting: C = B \cdot \log_2 (1 + SNR)

- But can’t measure SNR directly
  - Measure received signal strength (RSS), and noise N
  - RSS = S + N
  - For S>>N, SNR \sim RSS / N
Signal-to-Noise ratio

• Useful SNRs vary widely as capacity grows as log SNR
  – Thus SNR is usually given on a log scale

• Relative SNR units are deciBels (dB)
  – $\text{SNR} = 10 \log_{10} (\text{Signal} / \text{Noise})$
  – e.g., $3\text{dB} \approx 2$, $10\text{dB} = 10$, $20\text{dB} = 100$, $30\text{dB} = 1000$

• Absolute signal power levels are common too
  – $\text{dBm} = \text{deciBels relative to } 1\text{mW} (=0\text{dBm})$
  – e.g., $10\text{mW} = 10\text{dBm}$, $30\text{mW} = 12\text{dBm}$, etc.
SNR with Interference

• With wireless, receiver sees sum of all arriving signals
  – Plus thermal noise at the receiver (~100dBm for 802.11)

• Interfering signals (e.g., from neighboring cell) are intuitively like noise – they reduce our capacity to communicate

• Signal-to-interference-plus-noise ratio:
  – $\text{SINR} = \text{signal} / (\text{interference} + \text{noise})$
  – Appropriate metric when there is interference