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Inventory modelling for complex emergencies in humanitarian relief operations

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The increasing complexity and magnitude of global emergency relief operations create a critical need for effective and efficient humanitarian supply chain management processes. The irregular demand patterns and unusual constraints inherent in large-scale emergencies present unique challenges to physical supply systems. Indeed, the logistical needs of non-governmental organisations frequently surpass the capabilities of current emergency response approaches. There is only a limited body of research in humanitarian supply chain management, particularly in the area of inventory control. In this research, this limitation is addressed by developing a stochastic inventory control model that determines optimal order quantities and reorder points for a long-term emergency relief response.

Keywords: Inventory model; Emergency; Humanitarian logistics

1. Introduction

In 1998 there were 400 natural disasters reported by the International Federation of Red Cross and Red Crescent Societies (1999). These natural disasters affected more than 144 million people, resulting in 90,000 deaths and 5 million temporarily displaced persons. In 1998, the governmental donor community spent more than US$3 billion responding to the immediate effects of natural disasters and man-made emergencies (International Federation of Red Cross and Red Crescent Societies 1999). In 2005, the International Federation of the Red Cross and Red Crescent Societies (2005) reported that the number of disasters rose dramatically to an average of 707 disasters per year from 1999 to 2003, affecting an average of 213 million people per year.

The challenge for non-governmental organisations (NGOs) and other relief agencies responding to global emergencies (e.g. volcanic eruptions, earthquakes, floods, war) is how to prepare and manage relief activities in an unpredictable environment. Even with modern scientific advances, the onset of large-scale disasters can occur with little warning, and once a disaster strikes, the relief response often necessitates a variety of NGO expertise. Regardless of an NGO's operational speciality (e.g. housing and shelter, water and sanitation, pharmaceuticals) and whether the response is on a national or international level, each NGO shares the common critical objective of rapidly delivering the correct amount of goods, people and

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monetary resources to the needed locations. As a result, the ability of an NGO's supply chain and logistic operations directly affects the success of a relief effort.

Recent studies, such as Fenton (2003) and Thomas (2003), have often compared the current state of supply chain management capabilities within humanitarian organisations with that of the commercial sector in the 1970s and 1980s. At that time, the commercial sector began to realise the strategic advantages and significant improvements supply chain management could offer in effectiveness and efficiency. This led to extensive research in the area of supply chain and logistical analysis. Even so, quantitative methods and principles are rarely applied to humanitarian operations. Also, as global relief responses grow in scale and magnitude, humanitarian supply chain management is becoming increasingly complex. The growing logistical needs are outpacing the capabilities of current management approaches. This is partly due to the aid sector's regard for logistics as a necessary expense (rather than an important strategic component in their work), the lack of depth in operational knowledge (due to high humanitarian agency employee turnover) and the general lack of investment in technology and communication. The trends of increasing numbers of natural disasters and increasing emphasis on accountability have served as motivating factors to increase understanding and improve the capacity for delivering humanitarian relief. This work focuses specifically on developing an inventory management strategy for a warehouse supporting a complex emergency relief operation.

2. Literature review

Quantitative tools for emergency relief have typically come in the form of mathematical and network flow models. Oh and Haghani (1996) analysed the transportation of large amounts of many different commodities, such as food, clothing, medicine, medical supplies, machinery and personnel, in the most efficient manner to minimise the loss of life and maximise the efficiency of the rescue operations. The result was a formulation and solution of a multi-commodity, multi-modal network flow model for a generic disaster relief operation. The model was constructed to have four nodes, five arcs and three transportation modes. Two heuristics were developed, with the first utilising a Lagrangian relaxation approach and the second employing an iterative fix-and-run process. Oh and Haghani (1997) explored further their heuristic models and provided deeper, more detailed analysis.

As has been well documented in disaster relief analysis, the infrastructure for the transportation of relief supplies is often unreliable. Owing to this unreliability, Barbarosoglu *et al.* (2002) abandoned dependence on road networks and focused on the use of helicopters for aid delivery and rescue missions during natural disasters. Their research was mainly concerned with developing mathematical models that solve tactical and operational scheduling decisions regarding helicopter activities. The authors utilised existing research in helicopter routing to address crew assignment, routing and transportation issues encountered during the initial response phase of disaster management. Infrastructure unreliability as an obstacle to developing and maintaining supply chains was also the motivation behind Thomas's (2002) research. The author developed a method for quantifying the reliability of supply chains for contingency logistics systems based on reliability interference theory. Barbarosoglu and Arda (2004) further explored modelling the uncertainty involved in emergency response. They developed a two-stage stochastic programming framework for transportation planning in disaster response. In this work, the authors expanded on Oh and Haghani's (1996) deterministic multi-commodity, multi-modal network flow problem to include uncertainties that exist in estimating resource requirements of first-aid commodities, vulnerability of resource provider facilities and survivability of the connecting routes in the disaster area.

Ozdamar *et al.* (2004) examined logistics planning in emergencies involving the dispatch of commodities to distribution centres of affected areas. The network flow model developed by the author addresses a dynamic time-dependent transportation problem, and repetitively derives a solution at given time intervals to represent ongoing aid delivery. The model regenerates plans incorporating new requests for aid materials, new supplies, and transportation modes that become available during the current planning time horizon. The plan indicates the optimal mixed pick-up and delivery schedules for vehicles within the considered planning time horizon as well as the optimal quantities and types of loads to be picked up and delivered on these routes.

Sakakibara *et al.* (2004) also examined commodity flow in a road network during disaster response, and partitioned the road network into isolated components. The authors used a topological index to quantify road network accessibility, and then evaluated the isolation of districts in a disaster location. The authors presented a methodology for specifying effective road links for avoiding functional isolation of districts.

Mathematical modelling of inventory management in emergency relief efforts has received little attention in the literature. Depending on the nature of the disaster, humanitarian emergency operations can continue for many years. In order to ensure continuous capacity in long-term responses, NGOs often institute arrangements for the storage for relief items in warehouses located near the response location. This research focuses on developing an inventory management policy to improve the effectiveness and efficiency of emergency relief during long-term humanitarian responses.

3. System to be modelled: south Sudan relief efforts

The last decade has seen a decisive increase in the loss of life, property, and material damage due to the rising occurrence of natural disasters (United States Agency for International Development 2005). Also on the rise, and equally as devastating, are man-made disasters, which are referred to within humanitarian agencies as complex humanitarian emergencies (United States Committee for Refugees and Immigrants 2004). UNICEF (2003) defines a complex humanitarian emergency as “A humanitarian crisis in a country, region, or society where there is significant or total breakdown of authority resulting from internal or external conflict and which requires an international response that extends beyond the mandate or capacity of any single humanitarian agency”.

Complex humanitarian emergencies are typically rooted in racially, ethnically, or religiously charged warfare, and are frequently characterised by horrific violations of human rights. The disturbing trend with complex humanitarian emergencies is that when conflicts erupt within the borders of a country, the dividing line between civilians and combatants is frequently blurred. Militant or rebellious groups are usually the same civilians living and socialising in and around the villages they attack. This type of warfare commonly resorts to the use of insidious tactics where humanitarian agencies are denied access to groups of people in need of assistance. Also, at the centre of these shocking developments is the emergence of civilians, including woman, children and humanitarian workers, as the deliberate targets of warfare rather than its incidental victims. The vast devastation of complex humanitarian emergencies can also be attributed to the typically long length of the disaster, which imposes not only financial hurdles, but also legal, moral and political dilemmas on those attempting to provide relief. Humanitarian efforts responding to complex emergencies often last for many years, and therefore require logistics systems that can support long-term relief activities.

The destruction resulting from long-term complex humanitarian emergencies has recently been witnessed in the Balkans tragedy, the genocide in Rwanda and, most dramatically, in the civil war in south Sudan. The United States Holocaust Memorial Museum (2005) described the warfare in Sudan, Africa's largest country, as one of the most devastating humanitarian crises ever to affect the world. Since 1983, civil war has ravaged south Sudan, leaving over 2 million dead and over 4 million displaced (United States Committee for Refugees and Immigrants 2004). For over 20 years, humanitarian agencies have been responding to the emergency by airlifting aid to many parts of the underdeveloped south Sudan region. The United States Committee for Refuges and Immigrants (2004) estimated that millions of people are dependent on the relief supplies provided by humanitarian agencies. Air access is vital for agencies providing relief as the existing roads in Sudan are often in poor condition, heavily mined, prone to attack by bandits and militia, and at times impassable due to seasonal rains (United States Holocaust Memorial Museum 2005). The south Sudan civil war has been fought primarily with guerrilla warfare tactics, which are based on principles of ambush and sabotage. The result is a sporadic need for humanitarian aid in unpredictable quantities. Pre-positioning of relief supplies near the affected area has proven to be an effective strategy for responding to emergencies of this nature (UNICEF 2005).

In 1989, in response to the continual need for humanitarian aid in south Sudan, Operation Lifeline Sudan (OLS) was created under the co-ordination of the United Nations as a consortium of UNICEF, the World Food Programme, and more than 35 NGOs. The purpose of OLS was for the United Nations to provide the necessary air transport and security for NGO operations, and to provide a strict code of conduct in order to maintain high standards and impartiality in the delivery of humanitarian assistance to Sudanese civilians. OLS operates out of a United Nations base in the northwest Kenyan town of Lockichoggio, and provides more than a dozen daily airlifts of food, relief supplies and people (Humanitarian and War Project 2005). The air transport and security provided by the OLS have allowed many NGOs to pre-position relief items in warehouses throughout Lockichoggio. As such, Lockichoggio has developed into a logistical hub for the south Sudan relief efforts.

Global pre-positioning of relief supplies is an expansion of the warehousing strategy typically seen in complex humanitarian emergency responses. This response strategy allows NGOs to respond quickly to disasters with relief supplies from strategically stocked warehouses throughout the world. Global pre-positioning is a relatively new approach, and currently only a few NGOs can support the large expense of operating a warehouse that serves the international community. World Vision International has taken the lead in implementing a global pre-positioning system. Its global pre-positioning units (GPUs) are part of a strategy that allows it to respond rapidly and effectively to large-scale emergencies. The GPU system was initiated in 2000 with three warehouse locations (Denver, Colorado; Brindisi, Italy; Hanover, Germany), but the full impact of its operations has yet to be determined. The trade-off of operating a global pre-positioned system is rapid response but large transportation costs. The challenge for NGOs is to integrate a GPU system into a long-term humanitarian relief response effectively and efficiently.

The objective here is to develop an efficient, quick-response warehouse inventory policy for a humanitarian organisation responding to a complex humanitarian emergency. The analysis is based on a case study of a single humanitarian agency operating a warehouse in Lockichoggio, Kenya, and responding to the south Sudan crisis within the OLS framework.

4. Humanitarian relief inventory model

Quantitative multi-supplier supply chain inventory modelling is an active area of research [see, e.g. Minner and von-Guericke (2003) for a review of such models]. Our research develops a multi-supplier inventory model that accounts for the unique demand patterns that occur in humanitarian emergency relief operations. First, the relevant model framework will be provided, and then the model described in detail. Our model develops an inventory policy for the unique characteristics of a long-term complex humanitarian emergency relief response, considering the specific characteristics of the ordering process and demand distribution, yielding new expressions for on-hand inventory, stock-out probability, expected number of back-orders and, ultimately, the decision variables Q_1, r_1, Q_2 and r_2.

General multi-supplier inventory models, such as that of Moinzadeh and Nahmias (1988), assume a continuous review inventory system with two options for re-supply. These models are an extension of the standard (Q,r) inventory policy that allows for two different lot sizes (Q_1 and Q_2) and two different reorder levels (r_1 and r_2). An order of size Q_1 is placed when the inventory position reaches r_1 (this is a regular, or normal reorder option). An emergency reorder option is an expedited order of size Q_2 placed when the inventory reaches a position r_2 (where $r_1 > r_2$). The lead-times for normal and emergency reorders are assumed constant in the model and are represented as τ_1 and τ_2, respectively (where $\tau_1 > \tau_2$). In addition to requiring a shorter lead-time, the items ordered through the emergency re-supply channel are also assumed to incur higher fixed and per unit ordering costs than the normal orders. The rest of the notation is given in section 4.1, with units in parentheses, and will be used throughout the rest of the paper.

4.1 *Summary of modelling notation*

K_1: The fixed cost for placing a normal order (US\$)
K_2: The fixed cost for placing an emergency order (US\$)
c_1: The per unit cost for a normal order (US\$)
c_2: The per unit cost for an emergency order (US\$)
h: The inventory holding cost per item per unit time (US\$)
π: The back-order cost per item per unit time (US\$)
p: The probability of a stock-out occurring within a cycle
OH: The on-hand inventory per cycle (number of items)
BO: The number of back-ordered items per cycle (number of units)
T: The expected cycle length (time units)
μ: The expected demand rate (quantity per unit time)
D: The annual demand (number of units)
Q_1: The quantity of relief items in a normal reorder (units)
Q_2: The quantity of relief items in an emergency reorder (units)
r_1: The reorder level for normal orders (units)
r_2: The reorder level for emergency orders (units)

In this section, a multi-supplier inventory model is presented for the south Sudan relief operations. Data were collected on-site in Lockichoggio, Kenya, from interviews with warehouse managers and warehouse stock-keeping cards during September 2004. We develop the Sudan humanitarian inventory model as a continuous inventory review system with two options for re-supply. The model allows for two different lot sizes (Q_1 and Q_2) and two different reorder levels (r_1 and r_2). An order of size Q_1 is placed when the inventory position reaches r_1 (a regular, or normal re-supply option). The expedited emergency re-supply option is of size Q_2 and placed when the inventory reaches a position r_2. The normal and emergency re-supply options represent placing orders with Nairobi and an international supplier, respectively. The international supplier may be directly affiliated with the relief agency, as with the GPUs of World Vision, or may have no affiliation at all.

Our process for developing the model begins by analysing the expected reorder level for each cycle. Next, we focus on the on-hand inventory and expected number of back-orders, and compute the probability of a stock-out per cycle and the total cost equations. Finally, we derive optimality conditions.

4.2 *Assumptions*

The following assumptions will apply to the humanitarian relief model.

- $r_1 > r_2$, in a given cycle, an emergency order will never be placed before a normal order.
- $\tau_1 > \tau_2$, an emergency order has a shorter lead-time.
- The international supplier will not experience stock-outs.
- Replenishment lead-time for normal orders is 8 days, and for simplification is assumed to be constant.
- Items ordered through the emergency re-supply channel incur a higher fixed and per unit ordering cost than normal orders.
- Replenishment lead-time for emergency orders is 2 days, and due to relative low variation is assumed to be constant.
- $Q_1 > r_1$, an order quantity will be large enough not to trigger automatically an additional reorder.
- Demand ~U[1, b] units.
- Demand occurs in discrete 10-day intervals.

4.3 *Expected reorder level per cycle*

Humanitarian emergency relief systems require rapid response. Therefore, we assume that the inventory position is under continuous review. However, since the requests for items occur in discrete time intervals, the inventory policy shares some characteristics with periodic review models.

For the inventory model, let us define the following random variables.

R_e: The actual inventory level at the time when a reorder is placed, $R_e \leq r_1$
I: The inventory position just before a request is made, $I > r_1$
D: The demand ~U[1,b], where D and b are discrete, and $b \geq 1$
I': The inventory position just after a request has been fulfilled, $I' \in \Re$

In the humanitarian relief model, undershoots are possible (an undershoot is the difference between the set inventory reorder level and the actual inventory level when a reorder is placed).

Let the current inventory level be at a position i just before a request for d items is placed. Then, after the request has been fulfilled, the inventory level will drop to a position i'. Therefore, define i' as:

$$i' = i - d. \tag{1}$$

If $i' \leq r_1$, then a reorder must be placed, and the resulting reorder level, r_e, is $r_e = i'$. Let Y be the number of units below r_1 that the inventory level reaches when a reorder is placed in any given cycle. Then define the random variable Y as:

$$Y = \begin{cases} r_1 - I' & I' < r_1 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

and it follows that if $I' < r_1$ then:

$$R_e = r_1 - Y. \tag{3}$$

Then the expected value of the current inventory when the order is placed, $\mathrm{E}[R_e]$, follows from equation (3) as:

$$\mathrm{E}[R_e] = r_1 - \mathrm{E}[Y]. \tag{4}$$

We begin by analysing the current inventory position *I*, with a reorder level r_1, just before a demand *D* is placed. There are two cases for *I*.

Case 1: If $i > r_1 + b$, then i' cannot drop to r_1 or lower after the next request for items since the demand, *D*, is uniformly distributed and $\max(d) = b$. In this case, $i' > r_1$.

Case 2: If $i \leq r_1 + b$, then i' can drop to r_1 or lower after the next request for items. In this case, $i' \leq r$.

Therefore, in determining the expected reorder level, $\mathrm{E}[R_e]$, we only have to consider cases where reorders are possible. From the above, reorders are possible only when the current inventory level, *i*, is in the range $[r_1 + 1, r_1 + b]$. A plot of all possible values of Y, where $I \in [r_1 + 1, r_1 + b]$, is given in figure 1. *i*-Values, which represent the current inventory position before an order is received, are plotted along the *x*-axis. *y*-Values, which are the number of units below the reorder level r_1 when a reorder is placed, are plotted along the *y*-axis.

Now, let $h(y)$ be the probability density function of *Y*. Then, $h(y)$ is given by:

$$h(y) = \frac{b - y}{\sum_{n=1}^{b} n}, \quad 0 \leq y \leq b - 1.$$

Using the identity $\sum_{n=1}^{x} n = (x^2 + x)/2$ yields:

$$h(y) = \frac{2(b - y)}{(b^2 + b)}, \quad 0 \leq y \leq b - 1. \tag{5}$$

We can then find the expected value of Y, $\mathrm{E}[Y]$, from the expression: $\mathrm{E}[Y] = \sum_{y=0}^{b-1} y(h(y))$.

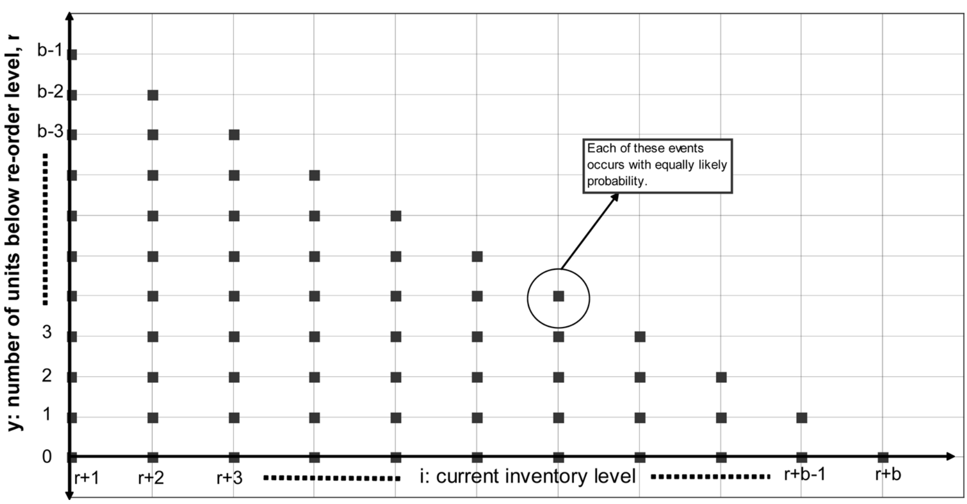

Figure 1. Distribution of *Y*.

It then follows that:

$$E[Y] = \sum_{y=0}^{b-1} \frac{2y(b-y)}{b^2+b}$$
$$= \frac{2}{b^2+b}\left(\sum_{y=0}^{b-1} yb - \sum_{y=0}^{b-1} y^2\right),$$

using the identity

$$\sum_{n=1}^{x} n^2 = \frac{(x^2+x)(2x+1)}{2},$$
$$= \frac{2}{b^2+b}\left(\frac{b\big((b-1)^2+(b-1)\big)}{2} - \frac{\big((b-1)^2+(b-1)\big)(2(b-1)+1)}{6}\right)$$
$$= \frac{b}{3} - \frac{1}{3}. \qquad (6)$$

Substituting equation (6) into equation (4) gives the expected reorder level, E[R_e], as:

$$\mathrm{E}[R_e] = r_1 - \left(\frac{b}{3} - \frac{1}{3}\right). \qquad (7)$$

4.4 *Expected average number of units held per cycle*

The fact that demands occur at discrete intervals has a significant effect on the amount of inventory held per cycle. This demand pattern causes the inventory level to experience periods of stability followed by sudden drops as items are removed in discrete batches. There are two distinct phases of the inventory cycle (phase I and phase II). Phase I is the replenishment lead-time of an order of size Q_1. Phase II is the time between the receipt of an order of size Q_1 and the placement of the next order. As long as the lead-time, τ_1, is less than the time between orders, then there will be no demand during the replenishment lead-time, τ_1, and the inventory level will remain constant until the arrival of Q_1.

For simplification, we shall approximate the discrete demand pattern of batches of items removed from inventory as continuous. The rate of demand is the expected demand per time, μ. The humanitarian relief inventory model is depicted in figure 2.

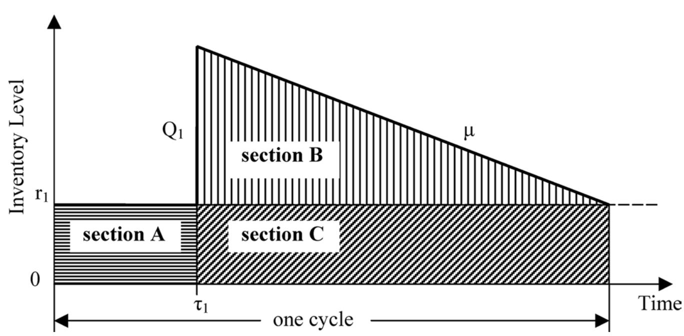

Figure 2. On-hand inventory plot for the humanitarian relief model with no emergency orders.

4.5 *Expected average number of units held per cycle (no emergency orders)*

The first situation to consider when determining the average amount of on-hand inventory is cycles with no emergency orders. When no emergency orders occur, the inventory level during phase I of the humanitarian relief model remains constant.

By dividing the graph in figure 2 into three separate sections (sections A, B and C), we can compute the total average on-hand inventory as the sum of each section. The area of section A is $r_1 * \tau_1$, the area of section B is $((1/2) * (Q_1/\mu) * Q_1)$ and the area of section C is $((Q_1/\mu) * r_1)$. The expression for OH_1 is the sum of these three terms and is given by:

$$OH_1 = \frac{Q_1^2}{2\mu} + \frac{Q_1 r_1}{\mu} + r_1 \tau_1. \quad (8)$$

Recall that when a reorder is placed, it is likely that the resultant inventory level will not be exactly equal to the reorder level r_1, but at the actual reorder level, r_e, where $r_1 > r_e$ (see figure 3).

We can again divide the total area under the curve of figure 4 into three separate sections (sections A, B and C) and compute the total on-hand inventory as the sum of each section. The area of section A is now $r_e * \tau_1$, the area of section B is $((1/2) * (r_e + Q_1 - r_1/\mu) * (r_e + Q_1 - r_1))$ and the area of section C is $((r_e + Q_1 - r_1/\mu) * r_1)$. OH_1 is the sum of these three terms, and is given by:

$$OH_1 = \frac{(R_e + Q_1 - r_1)^2}{2\mu} + \frac{(R_e + Q_1 - r_1)r_1}{\mu} + \tau_1 R_e. \quad (9)$$

4.6 *Expected average number of units held per cycle with emergency orders*

The second situation to consider when determining the average amount of on-hand inventory is cycles with emergency orders. Before we derive an expression for the average on-hand inventory when emergency orders are placed, we must first discuss the nature of a cycle with emergency orders. In the humanitarian relief model, an emergency order may be placed any time after a normal order is placed. There are two possible cases for the inventory level when an emergency order is placed:

Case 1: The inventory level is greater than or equal to zero ($i' \geq 0$).
Case 2: The inventory level has fallen below zero ($i' < 0$).

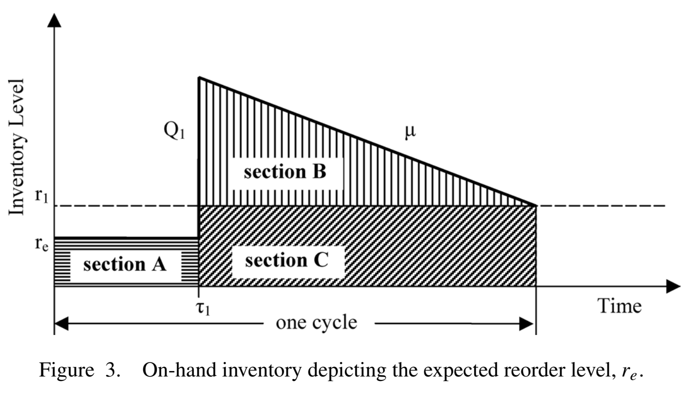

Figure 3. On-hand inventory depicting the expected reorder level, r_e.

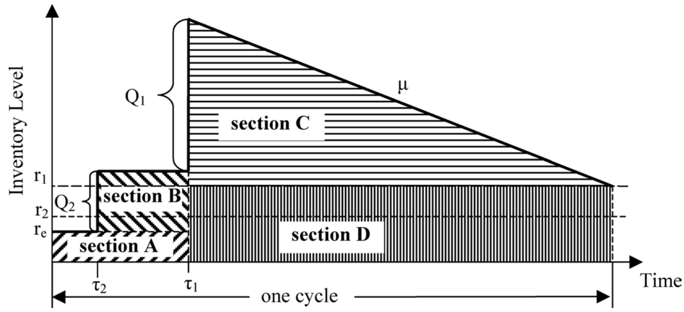

Figure 4. OH_2^*, a cycle in which an emergency order is placed with existing inventory.

Figure 4 depicts case 1, a cycle in which an emergency order is placed when the inventory level is greater than zero. We shall refer to the on-hand inventory in this case as OH_2^*.

To determine OH_2^*, we must find the area under the curve in figure 4. By dividing the curve into sections, we can take the same approach as before. The area of section A is $r_e * \tau_1$, the area of section B is $Q_1 * (\tau_1 - \tau_2)$, the area of section C is $((1/2) * ((r_e + Q_1 + Q_2 - r_1)/\mu) * (r_e + Q_1 + Q_2 - r_1))$ and the area of section D is $(((r_e + Q_1 + Q_2 - r_1)/\mu) * r_1)$. OH_2^* is the sum of these four terms and is given by:

$$OH_2^* = \frac{(R_e + Q_1 + Q_2 - r_1)^2}{2\mu} + \frac{(R_e + Q_1 + Q_2 - r_1)r_1}{\mu} + Q_1 * (\tau_1 - \tau_2) + \tau_1 R_e. \quad (10)$$

4.7 *Cycle length for relief cycles with and without emergency orders*

Now we can determine expressions for the cycle length T_1 (cycles without emergency orders) and for the cycle length T_2 (cycles with emergency orders). Equation (8) is the on-hand inventory for a cycle with no emergency orders. The total cycle length is the sum of the normal order lead-time (phase I) plus the expected length of time between a normal order arrival and the placement of the next normal order (phase II). The expected cycle length for cycles with no emergency orders, T_1, is given as:

$$T_1 = \tau_1 + \frac{(R_e + Q_1 - r_1)}{\mu}. \quad (11)$$

Equation (10) is the on-hand inventory for a cycle with emergency orders. As before, the total cycle length is the sum of the normal order lead-time (phase I) plus the expected length of time between normal order arrival and the placement of the next normal order (phase II). The expected cycle length for cycles with an emergency order, T_2, is then given as:

$$T_2 = \tau_1 + \frac{(R_e + Q_1 + Q_2 - r_1)}{\mu}. \quad (12)$$

4.8 *Revisiting: average number of units held per cycle with emergency orders*

We can now compute the average on-hand inventory per unit time for the cases of placing and not placing an emergency order to test whether it is ever cost-effective to place an emergency order while inventory still exits. Dividing equations (9) and (10) by their respective average cycle lengths, (11) and (12), we obtain expressions for the average on-hand inventory per unit time without emergency orders, AOH_1, and with emergency orders, AOH_2^*:

$$AOH_1 = \frac{[(R_e + Q_1 - r_1)^2/2\mu] + \{[(R_e + Q_1 - r_1)r_1]/\mu\} + \tau_1 R_e}{[(R_e + Q_1 - r_1)/\mu] + \tau_1} \tag{13}$$

$$AOH_2^* = \frac{\begin{array}{c}[(R_e + Q_1 + Q_2 - r_1)^2/2\mu] + \{[(R_e + Q_1 + Q_2 - r_1)r_1]/\mu\} \\ + Q_1{}^*(\tau_1 - \tau_2) + \tau_1 R_e\end{array}}{[(R_e + Q_1 + Q_2 - r_1)/\mu] + \tau_1}. \tag{14}$$

It can be shown that equation (13) will always be less than equation (14), therefore it is never economical to place an emergency order when the current inventory level is greater than zero. This does not mean that placing an emergency order will never be beneficial. We have just shown that, at the time of a reorder, if the inventory level is greater than or equal to zero, then it is not economical to place an emergency order, as it will only increase holding costs. However, if the inventory has fallen below zero (a stock-out occurs), then supplies have been back-ordered and an outstanding demand remains in the field. In this situation, it is reasonable to place an emergency order. Instead of waiting for the replenishment lead-time of τ_1, an emergency order can be placed and supplies delivered to the field within the time τ_2. Also, as we have shown, at any time the emergency order creates a positive inventory level, our holding costs increase. Therefore, when an emergency order is placed, it should be for exactly the amount of supplies back-ordered, which would bring our current inventory level to zero. Hence, for the humanitarian relief model we set Q_2 = expected number of back-orders, E[BO]. This cycle is depicted in figure 5.

The area under the curve in figure 5 represents the amount of on-hand inventory during a cycle with an emergency order. Since the amount back-ordered is never considered part of our on-hand inventory, our reorder level, R_e, is reduced to zero. Substituting $R_e = 0$ into equation (10) gives the following expression for on-hand inventory during a cycle with an emergency order:

$$OH_2 = \frac{(Q_1 - r_1)^2}{2\mu} + \frac{(Q_1 - r_1)r_1}{\mu}. \tag{15}$$

4.9 *Total average number of units held per cycle*

The total average on-hand inventory per cycle, OH, considers the probability of cycles with and without emergency orders. We define p as the probability that a cycle will have an

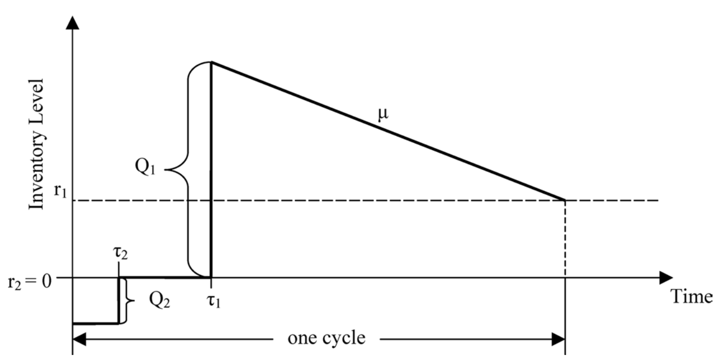

Figure 5. Realisation of a cycle in the humanitarian relief model with an emergency order.

emergency order. Then OH is given by:

$$OH = (1 - p)OH_1 + pOH_2. \tag{16}$$

Substituting the average on-hand inventory for cycles without emergency orders, equation (9), and the average on-hand inventory for cycles with emergency orders, equation (15), into equation (16) yields the following approximation for the average on-hand inventory during a cycle:

$$\begin{aligned} OH &= (1 - p)OH_1 + pOH_2 \\ &= (1-p)\left(\frac{(r_e + Q_1 - r_1)^2}{2\mu} + \frac{(r_e + Q_1 - r_1)r_1}{\mu} + \tau_1 r_e\right) \\ &\quad + p\left(\frac{(Q_1 - r_1)^2}{2\mu} + \frac{(Q_1 - r_1)r_1}{\mu}\right) \\ &= r_e(1-p)\left(\frac{Q_1}{\mu} + \tau_1\right) + \left(\frac{r_e^2(1-p) + Q_1^2 - r_1^2}{2\mu}\right). \end{aligned} \tag{17}$$

4.10 *Total average cycle length*

As in the calculation of the total average on-hand inventory in equation (16), the calculation for the total average cycle length considers the probability of both cycles with and without emergency orders. The total cycle length, T, is:

$$T = (1 - p)T_1 + pT_2. \tag{18}$$

The expected cycle length for cycles with an emergency order, T_2, as shown in figure 5, is:

$$T_2 = \tau_1 + \frac{(Q_1 - r_1)}{\mu}. \tag{19}$$

To determine the total expected cycle length, we substitute equations (15) and (11) into equation (14), which simplifies to:

$$T = \tau_1 + \frac{r_e(1-p) + Q_1 - r_1}{\mu}. \tag{20}$$

4.11 *Expected number of back-orders per cycle*

In order to determine the total cost per cycle due to back-orders, we must first determine the expected number of back-orders per cycle, E[BO]. Calculating E[BO] is very similar to calculating the expected reorder level, E[R_e]. In determining E[BO], we now consider the number of units below zero the inventory level reaches at the time a reorder is placed [that is, when y, the number of units below the reorder level, r_1, is greater than r_1, $(y > r_1)$]. Define Y' as the number of units below zero the inventory reaches when a reorder is placed. Then, $Y' = Y - r_1$, and the number of back-orders per cycle, BO, is given by $BO = Y'$, and E[BO] = E[Y'].

In calculating E[R_e], we determined the probability density function of Y, and that the range of Y was defined on $y \in [0, b - 1]$, where $y = 0$ describes a situation where the inventory position falls to exactly r_1 when a reorder is placed. Therefore, since we are now interested in

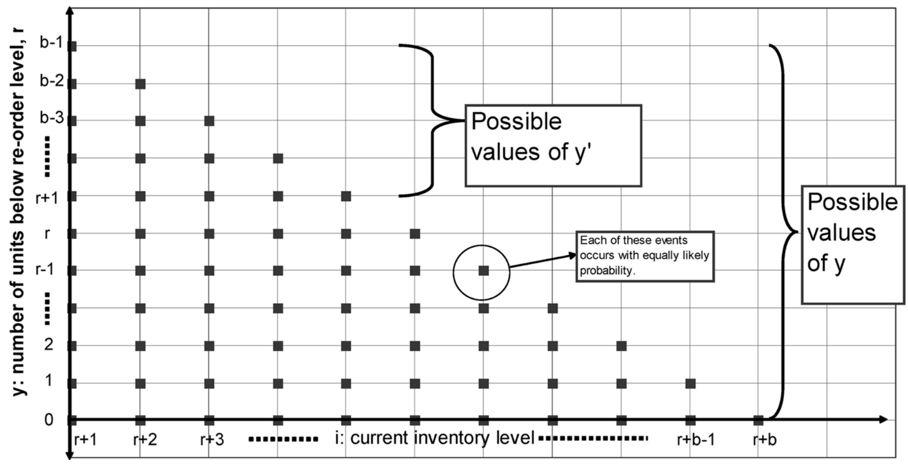

Figure 6. Probability density function of Y and Y'.

the situation in which $y > r_1$, our range for Y'' is defined as $Y' \in [r_1 + 1, b - 1]$, as shown in figure 6.

Since Y' is a subset of Y, we can take the probability density function of Y, originally given in equation (5), and limit its range to $r_1 + 1 \leq y \leq b - 1$. Substituting $y = y' + r_1$ into equation (5) and restricting the range allow us to compute the probability density function of Y', $h(y')$, as:

$$h(y') = \frac{2\left(b - r - y'\right)}{\left(b^2 + b\right)}, 1 \leq y' \leq b - r_1 - 1. \tag{21}$$

Then, $\mathrm{E}[Y'] = \sum_{y'=1}^{b-r_1-1} y'(h(y'))$, which can be computed as:

$$\begin{aligned} E[Y'] &= \sum_{y'=1}^{b-r_1-1} y'(h(y')) \\ &= \frac{2}{b^2+b}\left(\sum_{y=1}^{b-r_1-1} y'b - \sum_{y=1}^{b-r_1-1} y'r - \sum_{y=1}^{b-r_1-1} y^2\right) \\ &= \frac{2}{b^2+b}\left(\frac{b\left(\eta^2+\eta\right)}{2} - \frac{r_1\left(\eta^2+\eta\right)}{2} - \frac{\left(\eta^2+\eta\right)(2\eta+1)}{6}\right) \text{ where } \eta = b - r_1 - 1 \\ &= \frac{b^3 - 3b^2 r_1 + 3b r_1^2 - r_1^3 - b + r_1}{3(b^2+b)}. \end{aligned}$$

Therefore, the expected number of back-orders per cycle, $\mathrm{E}[BO]$, is:

$$\mathrm{E}[BO] = \frac{b^3 - 3b^2 r_1 + 3b r_1^2 - r_1^3 - b + r_1}{3(b^2+b)}. \tag{22}$$

4.12 *Per cycle back-order probabilities*

Previously, we defined p as the probability of an emergency order occurring within a cycle (and therefore a stock-out occurring within a cycle). We can now derive a value for p. We also defined Y' as the number of units below zero the inventory reaches when a reorder is placed, which is the number of back-orders, and derived the probability density function of Y', $h(y')$, in equation (5). Therefore, p, the probability of a stock-out in a cycle, is the sum of the individual probability densities of Y', where $Y' \geq 1$. Therefore:

$$\begin{aligned}
p &= \sum_{y'=1}^{b-r_1-1} h(y') \\
&= \sum_{y'=1}^{b-r_1-1} \frac{2(b-r_1-y')}{(b^2+b)} \\
&= \frac{2}{(b^2+b)}\left[\sum_{y'=1}^{b-r_1-1} b - \sum_{y'=1}^{b-r_1-1} r_1 - \sum_{y'=1}^{b-r_1-1} y'\right] \\
&= \frac{2}{(b^2+b)}\left[b(b-r_1-1) - r_1(b-r-1) - \left(\frac{(b-r_1-1)^2+(b-r_1-1)}{2}\right)\right] \\
&= \frac{b^2-2br_1+r_1^2-b+r_1}{(b^2+b)}. \qquad (23)
\end{aligned}$$

4.13 *Total cost equation*

The total cost is computed on a per cycle basis, where one cycle is defined as the time between consecutive orders of Q_1. The total cost is the sum of the fixed cost of placing a normal order, K_1, the per unit cost for a normal order, c_1Q_1, the fixed and per unit cost of an emergency order multiplied by the probability of placing an emergency order, $p(K_2+c_2Q_2)$, the inventory holding costs, $h[OH]$, and the back-order costs, $\pi\mathrm{E}[BO]$. Therefore, the total cost in a cycle, TC, is given as:

$$TC(Q_1, Q_2, r_1, r_2) = K_1 + c_1Q_1 + p(K_2 + c_2Q_2) + h[OH] + \pi E[BO]. \qquad (24)$$

It then follows that the average cost per unit time, AC, is:

$$AC(Q_1, Q_2, r_1, r_2) = \frac{TC(Q_1, Q_2, r_1, r_2)}{T}. \qquad (25)$$

4.14 *Deriving optimal conditions*

We can now derive optimal conditions. For fixed values of r_1 and r_2, the optimal values of Q_1 and Q_2 can be found using the optimality conditions for the average cycle cost. The optimality conditions are:

$$\frac{\partial AC}{\partial Q_1} = \frac{\partial AC}{\partial Q_2} = 0 \qquad (26)$$

Recall that an emergency order of Q_2 is placed to raise the inventory level back to zero. Therefore, Q_2 is the expected number of back-orders in a cycle, so $Q_2 = \mathrm{E}[BO]$. Substituting $Q_2 = \mathrm{E}[BO]$ into the total cost equation (24), gives:

$$TC(Q_1, r_1, r_2) = K_1 + c_1Q_1 + p(K_2 + c_2E[BO]) + h[OH] + \pi\mathrm{E}[BO]. \qquad (27)$$

Since the expected number of back-orders, equation (22), and the total average on-hand inventory, equation (17), are independent of Q_2, the total cost equation (24) is independent of Q_2.

The average total cycle length, equation (20), is also independent of Q_2, therefore the partial derivative in equation (26) with respect to Q_2 is equal to zero. This reduces the number of optimality equations to one, which can be solved directly for Q_1. The optimality condition is as follows:

$$\frac{\mathrm{d}AC}{\mathrm{d}Q_1} = 0$$

where

$$\frac{\mathrm{d}AC}{\mathrm{d}Q_1} = -\frac{TC}{T^2}\frac{\mathrm{d}T}{\mathrm{d}Q_1} + \frac{1}{T}\frac{\mathrm{d}TC}{\mathrm{d}Q_1}. \tag{28}$$

Solving equation (28) gives the following closed-form expression for Q_1 (see the Appendix for derivation):

$$Q_1 = r_1 + r_e(1-p) - \tau_1\mu \pm \frac{1}{h}\sqrt{\begin{array}{c} h^2\left(r_e\left(2\tau_1\mu\left(1-p\right)+pr_e\left(1-p\right)\right)+\tau_1\mu\left(2r_1+\tau_1\mu\right)\right)+ \\ 2\,h\mu\left(c_1\left(r_e\left(1-p\right)+r_1-\mu\tau_1\right)+K_1+pK_2+E[BO]\left(pc_2+\pi\right)\right)\end{array}}. \tag{29}$$

4.15 *Solution procedure*

The following steps outline the procedure for developing a solution to the model.

(1) Select an order stock-out risk (OSOR). The OSOR is the risk we are willing to accept that any cycle in our system will experience a stock-out, and will depend on the severity of the emergency. Intuitively, the more critical the aid, the lower the OSOR we shall select. We can set the OSOR as we wish since the stock-out probability, equation (23), is independent of Q_1.
(2) Set $p =$ OSOR and determine the reorder level r_1 from equation (23).
(3) Use r_1 to determine the expected reorder level, E[R_e], from equation (4).
(4) Determine the expected number of back-orders per cycle, E[BO], using r_1 in equation (22).
(5) Substitute r_1, r_e, p, E[BO] and the initial parameters into equation (29) and solve for Q_1

5. Conclusions and future research

In this research, our objective was to develop an inventory model for a pre-positioned warehouse responding to a complex humanitarian emergency, which is one of the three emergency classifications to which humanitarian and non-governmental agencies respond (rapid onset and slow onset are the other two). Complex humanitarian emergencies are unique due to their unpredictable demand patterns and long durations. The high (often life-threatening) stakes of humanitarian relief place a heavy emphasis on quick logistics response. It is essential that humanitarian logistics operations (including warehousing in a complex emergency) be performed as efficiently and effectively as possible. Field research for this paper was performed with our host organisation, World Vision International. Data were collected from its warehouse operations in Lockichoggio, Kenya, as it was responding to the complex humanitarian emergency in south Sudan. Based on our recorded data, we developed a mathematical model that optimised the reorder quantity and reorder level based on reordering, holding and back-order costs.

The research presented in this paper is a first step in developing strategic inventory management systems for humanitarian relief. The model investigated a single "item" (which may be interpreted as a single type of relief kit or a single set of items), developed order quantities that were independent of vehicle or container sizes, and assumed a continuous demand approximation. Future work would investigate the effects of a (correlated) multi-item system that incorporates container sizes, which would probably prove especially important in a relief situation. Another step would include explicit modelling of the discrete inventory removals (rather than using a continuous approximation). Additionally, a more thorough study to understand the full implications of back-order costs within humanitarian logistics is needed. The back-order cost represents a penalty for unmet demand, but since there are no financial profits made in humanitarian logistics, the lost sales approach used in commercial logistics is not necessarily appropriate. A back-order cost in humanitarian logistics represents potential suffering (or loss of life) endured by a potential recipient from not receiving a relief item, but also for the "advertising" opportunity lost by the relief agency for not delivering an item. NGOs do not have an income stream to fund their work outside donations, and therefore must remain vigilant to ensure their actions are being noticed within the donor community. The back-order cost used in our model is a one-time penalty applied to each unit of demand that could not be met immediately from inventory. Eventually, back-orders are met from an expedited order, but the length of time required to fulfil back-orders is not considered. In the real system, there is a window of opportunity during which relief items can be used in the field, and the longer the delay for a needed item, the more devastating the effects. Future research would analyse the impact of time-dependent shortage costs and develop a more detailed quantification methodology for assessing shortages in humanitarian logistics.

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Appendix: Closed-form expression for Q_1

Given the optimality condition

$$\frac{\mathrm{d}AC}{\mathrm{d}Q_1} = 0$$

where

$$AC(Q_1, Q_2, R_1, R_2) = \frac{TC(Q_1, Q_2, R_1, R_2)}{T},$$
$$TC(Q_1, Q_2, r_1, r_2) = K_1 + c_1 Q_1 + p(K_2 + c_2 E[BO]) + h[OH] + \pi E[BO],$$
$$T = \tau_1 + \frac{R_e(1-p) + Q_1 - r_1}{\mu}$$

and

$$OH = r_e(1-p)\left(\frac{Q_1}{\mu} + \tau_1\right) + \left(\frac{r_e^2(1-p) + Q_1^2 - r_1^2}{2\mu}\right)$$

we can form the expression for AC as:

$$AC = \frac{K_1 + c_1 Q_1 + p(K_2 + c_2 E[BO]) + h\begin{pmatrix} R_e(1-p)((Q_1/\mu) + \tau_1) \\ +(R_e^2(1-p) + Q_1^2 - r_1^2)2\mu \end{pmatrix} + \pi E[BO]}{\tau_1 + (R_e(1-p) + Q_1 - r_1)/2\mu}.$$

We can factor and simplify AC to get the following:

$$AC = \frac{-1}{2(\tau_1\mu + R_e p + Q_1 - r_1)}(-2K_1\mu - 2c_1 Q_1 \mu - 2p\mu K_2 - 2p\mu c_2 E[BO] - 2hR_e Q_1 - 2hR_e\tau_1\mu + 2hR_e p Q_1 + 2hR_e p\tau_1\mu - hR_e^2 + hR_e^2 p - hQ_1^2 + hr_1^2 - 2\pi E[BO]\mu).$$

Taking the derivative of AC with respect to Q_1 gives:

$$\frac{\mathrm{d}AC}{\mathrm{d}Q_1} = -\frac{-2c_1\mu - 2hR_e + 2hR_e p - 2hQ_1}{2(\tau_1\mu + R_e - R_e p + Q_1 - r)} + \frac{1}{2(\tau_1\mu + R_e - R_e p + Q_1 - r_1)^2} \times (-2K_1\mu - 2c_1 Q_1\mu - 2p\mu K_2 - 2p\mu c_2 E[BO] - 2hR_e Q_1 - 2hR_e\tau_1\mu + 2hR_e p Q_1 + 2hR_e p\tau_1\mu - hR_e^2 + hR_e^2 p - hQ_1^2 + hr_1^2 - 2\pi E[BO]\mu).$$

Setting $\mathrm{d}AC/\mathrm{d}Q_1 = 0$ yields the following expression:

$$\frac{-2c_1\mu - 2hr_e + 2hr_e p - 2hQ_1}{2(\tau_1\mu + r_e - r_e p + Q_1 - r)} = \frac{1}{2(\tau_1\mu + r_e - r_e p + Q_1 - r)^2} \times (-2K_1\mu - 2c_1 Q_1\mu - 2p\mu K_2 - 2p\mu c_2 E[BO] - 2hr_e Q_1 - 2hr_e\tau_1\mu + 2hr_e pQ_1 + 2hr_e p\tau_1\mu - hr_e^2 + hr_e^2 p - hQ_1^2 + hr^2 - 2\pi E[BO]\mu).$$

Solving for Q_1 yields:

$$Q_1 = r_1 + R_e(1-p) - \tau_1\mu \pm \frac{1}{h}\sqrt{\begin{array}{l} h^2(R_e(2\tau_1\mu(1-p) + pR_e(1-p)) \\ +\tau_1\mu(2r_1 + \tau_1\mu)) + 2h\mu(c_1(R_e(1-p) + r_1 - \mu\tau_1) \\ +K_1 + pK_2 + E[BO](pc_2 + \pi)) \end{array}}.$$